recall Thm 3.5

if $v_1, \ldots, v_n$ is a basis of V and $T: V \to W$ is linear

then the values of $Tv_1, \ldots, Tv_n$ determine the values

of T on arbitrary vectors in V.

$Tv_1, \ldots, Tv_n$ 决定了 Tv $\forall v \in V$.

Def matrix

$$A = \begin{pmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & & \vdots \\ A_{m1} & \cdots & A_{mn} \end{pmatrix}$$

☆3.32 Def   Suppose $T \in \mathcal{L}(V, W)$ and

$v_1, \ldots, v_n$ is a basis of V and

$w_1, \ldots, w_m$ is a basis of W.

The matrix of T with respect to these bases

is the $m \times n$ matrix $\mathcal{M}(T)$

whose entries A_{jk} are defined by

$$Tv_k = A_{1k} w_1 + \cdots + A_{mk} w_m$$

insight: $Tv_k \in W$, W 中任一向量可由 W 的一个基线性表出.
                    (represent as a linear combination)

3.33 example Suppose $T \in \mathcal{L}(\mathbb{F}^2, \mathbb{F}^3)$

is defined by $T(x,y) = (x+3y, \; 2x+5y, \; 7x+9y)$

Find the matrix of T with respect to the

standard bases of $\mathbb{F}^2$ and $\mathbb{F}^3$

Solution
$$T(1,0) = (1, \; 2, \; 7)$$
$\qquad e_1 = (1,0) \qquad A_{11} \quad A_{21} \quad A_{31}$

$$T(0,1) = (3, \; 5, \; 9)$$
$\qquad e_2 = (0,1) \qquad A_{12} \quad A_{22} \quad A_{32}$

$$M(T) = \begin{pmatrix} 1 & 3 \\ 2 & 5 \\ 7 & 9 \end{pmatrix}$$