R^n and C^n

2020年7月5日 19:11

Complex Numbers.

> 曾经被这样一条古老的状态刷屏了：有人问一个法国四年级小学生:3+4等于几？回答：不知道。问：那4+3等于几？还是回答：不知道。问：那你小学都学了些什么呀？答：我知道3+4==4+3。问：为什么呀？答：因为加法构成一个Abel群啊。

↳ commutativity.

$\forall x, y, z \in \mathbb{R}$

commutativity $\quad x+y=y+x$ and $xy=yx$

associativity $\quad x+(y+z)=(x+y)+z$ and $x(yz)=(xy)z$

identities $\quad x+0=x$ and $x\cdot 1=1\cdot x=x$

additive inverse $\quad \forall x \in \mathbb{R}$ there exists a unique $y \in \mathbb{R}$ s.t. $x+y=0$

multiplicative inverse $\quad \forall x \in \mathbb{R}$ there exists a unique $y \in \mathbb{R}$ s.t. $xy=1$

distributive $\quad z(x+y)=zx+zy = (x+y)z.$

reference: Mathematical Analysis, Zorich

Section 2.1 Axiom System and some General Properties of the Set of Real Numbers

Principles of Mathematical Analysis

1.1 Def Complex numbers. $\quad i^2=-1$

- A complex number is an ordered (a,b), where $a, b \in \mathbb{R}$
  we will write as $a+bi$
- The set of all complex numbers:
  $$\mathbb{C}=\{a+bi : a, b \in \mathbb{R}\}$$
- Addition and Multiplication on $\mathbb{C}$.
  $$(a+bi)+(c+di)=(a+c)+(b+d)i$$
  $$(a+bi)(c+di)=(ac-bd)+(ad+bc)i$$

1.3 Properties of complex arithmetic.

$\forall x, y, z \in \mathbb{C}$

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distributive $\quad z(x+y)=zx+zy$

ex 4–9.

4. Show $\alpha+\beta=\beta+\alpha$ for all $\alpha, \beta\in\mathbb{C}$.

$\alpha=a+bi \qquad a, b, c, d \in\mathbb{R}$

$\beta=c+di$

$$\alpha+\beta=(a+c)+(b+d)i$$

$$=(c+a)+(d+b)i \quad \text{commutativity.}$$

$$=\beta+\alpha \quad \text{definition of complex numbers.}$$

1.5 Let $\alpha, \beta\in\mathbb{C}$.

- let $-\alpha$ denote the additive inverse of α.

  $\alpha+(-\alpha)=0.$

- $\beta-\alpha=\beta+(-\alpha)$

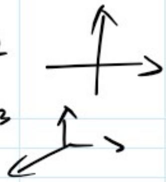
- $\alpha \beta = 1.$ $\quad \alpha(1/\alpha) = 1$

$\beta := \frac{1}{\alpha}.$

1.6 $\mathbb{F}$

denotes for either $\mathbb{R}$ or $\mathbb{C}$.

$\mathbb{R}^n, \mathbb{C}^n.$ $\quad \mathbb{R}^2$ $\quad \mathbb{R}^3$



$\mathbb{R} \times \mathbb{R}$

$\mathbb{R} \times \mathbb{R} \times \mathbb{R}.$

cartesian product.

$\mathbb{R}^n = \{ (x_1, \ldots, x_n) : x_1, \ldots, x_n \in \mathbb{R} \}$


René Descartes (/deɪˈkɑːrt/ or UK: /ˈdeɪkɑːrt/; French: [ʁəne dekaʁt] listen); Latinized: **Renatus Cartesius**;
February 1650[15][16][17][18]:58) ... tist. A native of the King
about 20 years (1629–1649) of ... e in the Dutch States Ar
Prince of Orange and the Stadt ... able intellectual figures
Descartes is also widely regard

Many elements of Descartes's ... he revived Stoicism of th
philosophers like Augustine. In ... on two major points: firs
corporeal substance into matter ... nds, divine or natural, ir
phenomena.[20] In his theology, ... reation. Refusing to acc

Latinisation of names, also known as **onomastic Latinisation**, is the practice of rendering a *non*-Latin name in a Latin style. It is commonly found with historical proper names, including personal names and toponyms, and in the standard binomial nomenclature of the life sciences. It goe

1.8 Def. List.

$n \in \mathbb{N}^+$

Suppose n is a nonnegative integer.

A list of length n is an ordered collection of n elements.

$(x_1, \ldots, x_n).$

$(x_1, x_2) \neq (x_2, x_1)$

$\{x_1, x_2\} = \{x_2, x_1\}$

$(x_1, x_2, \ldots)$ list?

$\left(1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots\right)$ is not a list.

$\mathbb{F}^n = \{ (x_1, \ldots, x_n) : x_j \in \mathbb{F} \text{ for } j = 1, \ldots, n \}.$

x_j is the j^{th} coordinate

x_j is the jth coordinate of $(x_1, \ldots, x_n)$.

$\mathbb{F}^n$

$$(x_1, \ldots, x_n) + (y_1, \ldots, y_n) = (x_1 + y_1, \ldots, x_n + y_n)$$

1.13 If $x, y \in \mathbb{F}^n$, then $x + y = y + x$.

Pf. Suppose $x = (x_1, \ldots, x_n)$
$y = (y_1, \ldots, y_n)$

$$\begin{aligned} x + y &= (x_1, \ldots, x_n) + (y_1, \ldots, y_n) \\ &= (x_1 + y_1, \ldots, x_n + y_n) && \text{Commutativity of } \mathbb{F} \\ &= (y_1 + x_1, \ldots, y_n + x_n) \\ &= (y_1, \ldots, y_n) + (x_1, \ldots, x_n) && \text{definition of addition in } \mathbb{F}^n. \\ &= y + x \end{aligned}$$

$\mathbb{F}^n$.
$(x_1, \ldots, x_n)$, $x_j \in \mathbb{F}$.

$$(x_1, \ldots, x_n) + \boxed{} = (x_1, \ldots, x_n).$$

$$\underbrace{(0, \ldots, 0)}_{\shortparallel}$$

1.15 example.

$x + 0 = x$ for all $x \in \mathbb{F}^n$

$0 \in ?$ addition in $\mathbb{F}^n$.

$0 \in \mathbb{F}^n$.

$0 \in \mathbb{F}$

1.16 Def

$x = (x_1, \ldots, x_n) + \boxed{} = 0 \in \mathbb{F}^n$

$-x = (-x_1, \ldots, -x_n)$.

$\mathbb{F}$. $+$, $\times$

$\mathbb{F}^n$ $+$, $\times$?

1.17. Scalar multiplication

The product of number λ and a vector in $\mathbb{F}^n$:

$$\lambda \cdot (x_1, \ldots, x_n) = (\lambda x_1, \ldots, \lambda x_n).$$

$\lambda \in \mathbb{F}$, $(x_1, \ldots, x_n) \in \mathbb{F}^n$.

$$\lambda (x_1, \ldots, x_n) = (\lambda x_1, \ldots, \lambda x_n)$$

$\mathbb{F}$ $\mathbb{F}$