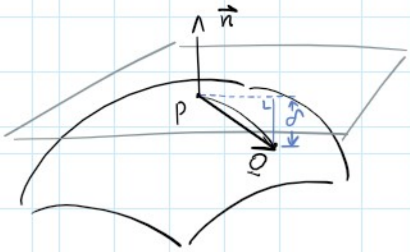
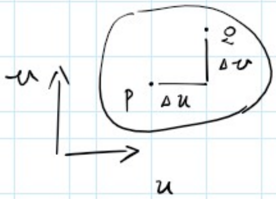
第二基本形式

2020年10月26日 16:10

1. 引入.

$\Sigma: \vec{r} = \vec{r}(u, v)$, $(u, v) \in D$





$$\vec{PQ} = \vec{r}(u+\Delta u, v+\Delta v) - \vec{r}(u, v)$$
$$= \vec{r}_u(u,v)\Delta u + \vec{r}_v(u,v)\Delta v + \frac{1}{2}\left(\vec{r}_{uu}(\Delta u)^2 + 2\vec{r}_{uv}\Delta u\Delta v + \vec{r}_{vv}(\Delta v)^2\right) + \varepsilon$$

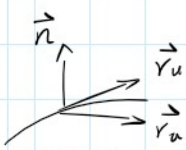
$$\delta = \vec{PQ}\cdot\vec{n} = \frac{1}{2}\left[(\vec{r}_{uu}\cdot\vec{n})(\Delta u)^2 + 2(\vec{r}_{uv}\cdot\vec{n})\Delta u\Delta v + (\vec{r}_{vv}\cdot\vec{n})(\Delta v)^2\right] + \varepsilon$$

$$L = \vec{r}_{uu}(u,v)\cdot\vec{n}(u,v)$$
$$M = \vec{r}_{uv}(u,v)\cdot\vec{n}(u,v)$$
$$N = \vec{r}_{vv}(u,v)\cdot\vec{n}(u,v)$$

$$\mathrm{II} = L(du)^2 + 2M\,du\,dv + N(dv)^2$$

2. 另一种表达方式

$$\vec{n} = \frac{\vec{r}_u \times \vec{r}_v}{|\vec{r}_u \times \vec{r}_v|}$$



↑只要想起这张图

$\vec{n}\cdot\vec{r}_u = 0$, $\vec{n}\cdot\vec{r}_v = 0$

$\Downarrow$ differentiate over u, v

$$\begin{cases} \vec{n}_u\cdot\vec{r}_u + \vec{n}\cdot\vec{r}_{uu} = 0 \\ \vec{n}_v\cdot\vec{r}_u + \vec{n}\cdot\vec{r}_{uv} = 0 \end{cases}, \quad \begin{cases} \vec{n}_u\cdot\vec{r}_v + \vec{n}\cdot\vec{r}_{vu} = 0 \\ \vec{n}_v\cdot\vec{r}_v + \vec{n}\cdot\vec{r}_{vv} = 0 \end{cases}$$

$\Rightarrow$ $\vec{n}\cdot\vec{r}_{uu} = -\vec{n}_u\cdot\vec{r}_u$, $\vec{n}\cdot\vec{r}_{uv} = -\vec{n}_v\cdot\vec{r}_u = -\vec{n}_u\cdot\vec{r}_v$, $\vec{n}\cdot\vec{r}_{vv} = -\vec{n}_v\cdot\vec{r}_v$

$L = -\vec{n}_u\cdot\vec{r}_u$, $M = -\vec{n}_v\cdot\vec{r}_u = -\vec{n}_u\cdot\vec{r}_v$, $N = -\vec{n}_v\cdot\vec{r}_v$

$$d\vec{r} = \vec{r}_u du + \vec{r}_v dv$$
$$d\vec{n} = \vec{n}_u du + \vec{n}_v dv$$
$$\Rightarrow d\vec{n}\cdot d\vec{r} = (\vec{r}_u du + \vec{r}_v dv)\cdot(\vec{n}_u du + \vec{n}_v dv)$$
$$= \vec{r}_u\cdot\vec{n}_u(du)^2 + (\vec{r}_u\cdot\vec{n}_v + \vec{r}_v\cdot\vec{n}_u)\,du\,dv + (\vec{r}_v\cdot\vec{n}_v)(dv)^2$$
$$= -\left(L(du)^2 + 2M\,du\,dv + N(dv)^2\right)$$

$$= -\left(L(du)^2 + 2M\,du\,dv + N(dv)^2\right)$$

$$\Rightarrow \mathrm{II} = -d\vec{n} \cdot d\vec{r}$$